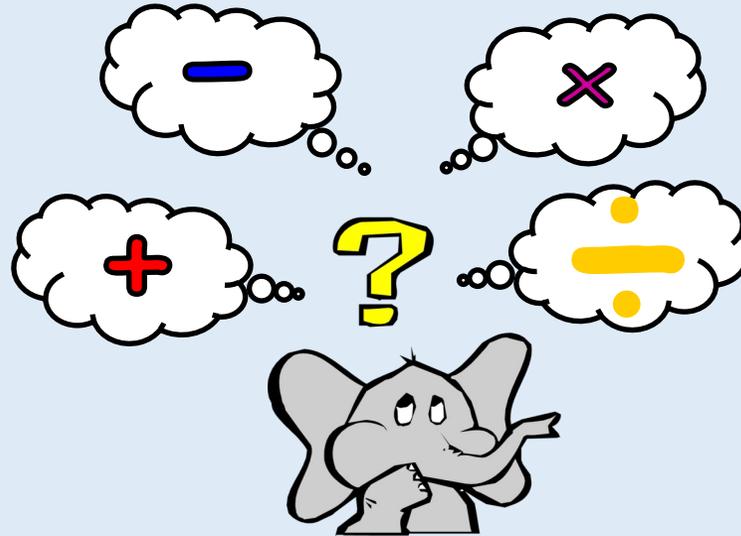
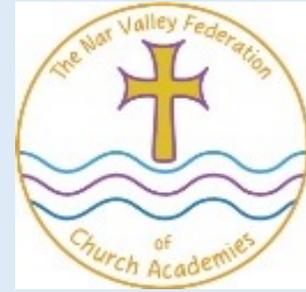


The Nar Valley Mathematics Calculation Policy: Part 2 Multiplication and Division



Commissioned by The PiXL Club Ltd.
June 2016

ADAPTED FOR USE AT THE NAR VALLEY FEDERATION OF SCHOOLS BY E. WILLGRESS – JANUARY 2020

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About Nar Valley's PiXL Calculation Policy



- The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

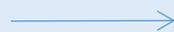
- Age stage expectations:

The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014 and the method(s) shown for each year group should be modelled to the vast majority of pupils. However, it is vital that pupils are taught according to the pathway that they are currently working at and are showing to have 'mastered' a pathway before moving on to the next one. Of course, pupils who are showing to be secure in a skill can be challenged to the next pathway as necessary.

- Choosing a calculation method:

Before pupils opt for a written method they should first consider these steps:

Can I do it in my head using a mental strategy?



Could I use some jottings to help me?



Should I use a formal written method to work it out?

Calculation Guidance Principles

- *Develop children's fluency with basic number facts*
- *Develop children's fluency in mental calculation*
- *Develop children's understanding of the = symbol*
- *Teach inequality alongside teaching equality*
- *Use empty box problems*
- *Use intelligent practice*
- *Expose mathematical structure and work systematically*
- *Move between the concrete and the abstract*
- *Contextualise the mathematics*



Concrete resources

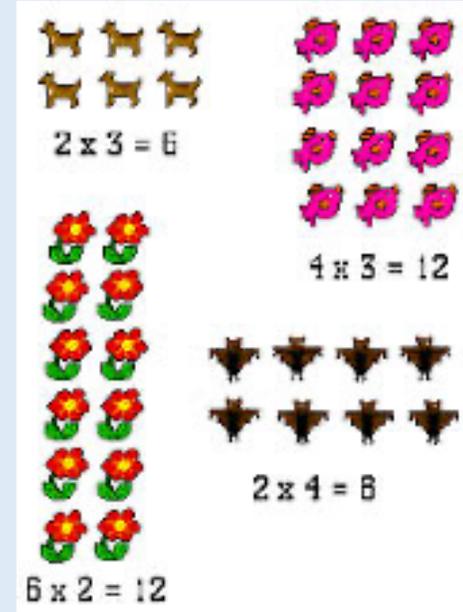
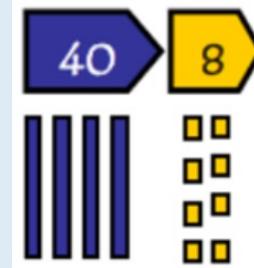
- Place value counters
- Dienes
- Place value charts
- Arrays
- Multiplication squares
- 100 square
- Number lines
- Blank number lines
- Counting stick



Multiplication



1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100



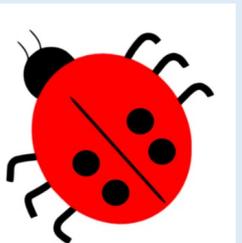
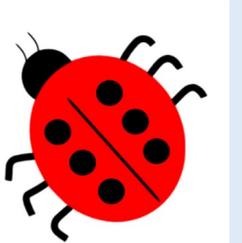
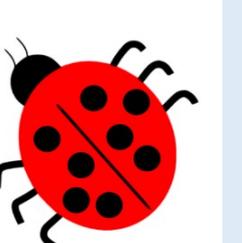
multiplication **product**
 once, twice, three times
double **groups of**
 repeated addition **lots of**
 array, row, column **multiply**
times **multiple**



Multiplication: Reception

Early learning goal statutory requirement:
✓ They solve problems, including doubling, halving and sharing.

Use pictorial representations and concrete resources to double numbers to 10.

			
$1 + 1 = 2$	$2 + 2 = 4$	$3 + 3 = 6$	$4 + 4 = 8$

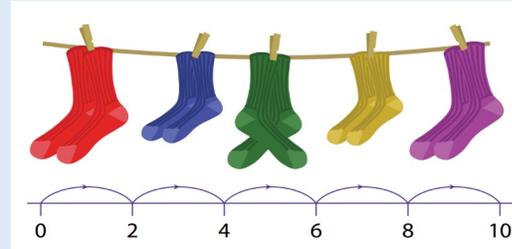
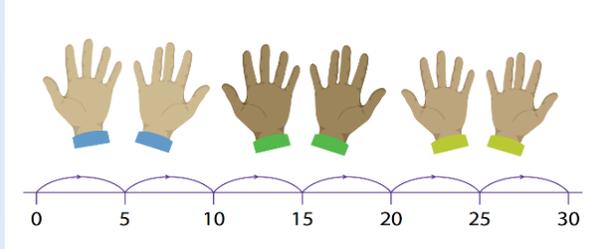
Use concrete sources, role play, stories and songs to begin counting in twos, fives and tens.

Multiplication: Year 1

Year 1 statutory requirement:

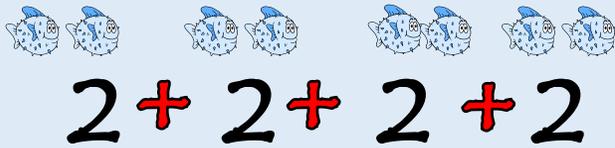
✓ Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Count in twos, fives and tens using practical resources, role play, stories and songs.



Use arrays

Understand multiplication as repeated addition – use concrete objects to support understanding.

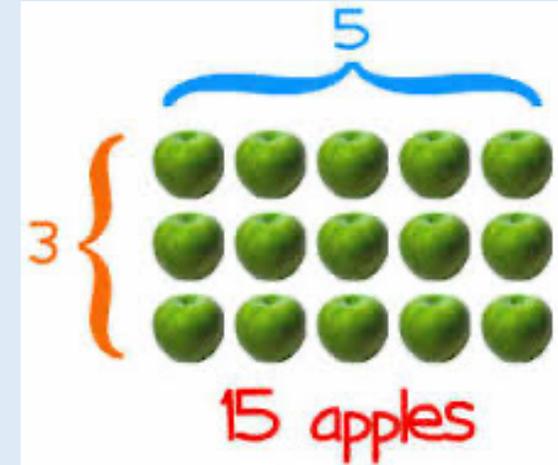
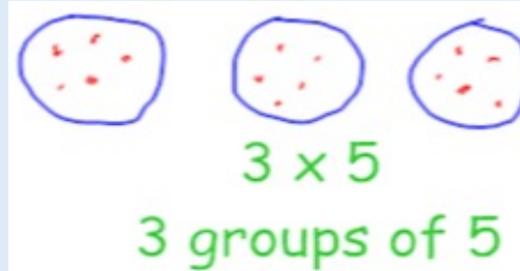


$$5 + 5 + 5$$

or

$$3 \times 5$$

Use pictorial representations



Multiplication: Year 2



Year 2 statutory requirement:

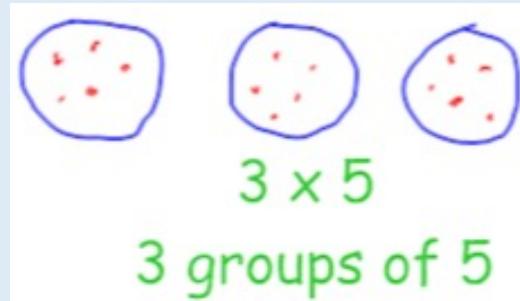
- ✓ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs.
- ✓ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Further develop understanding of multiplication as repeated addition.

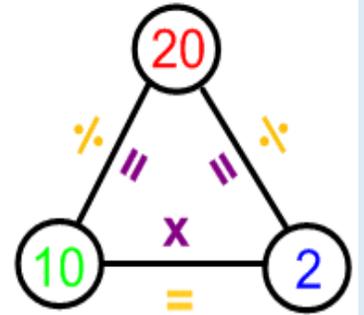
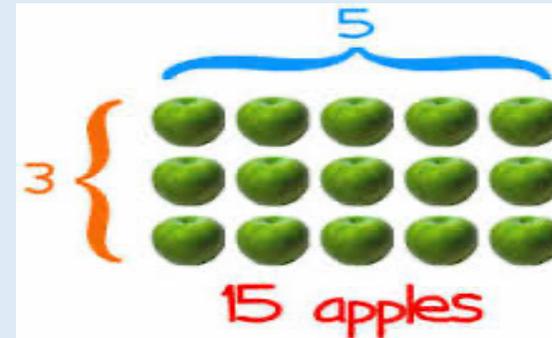


$5 + 5 + 5$
or
 5×3

Use pictorial representations

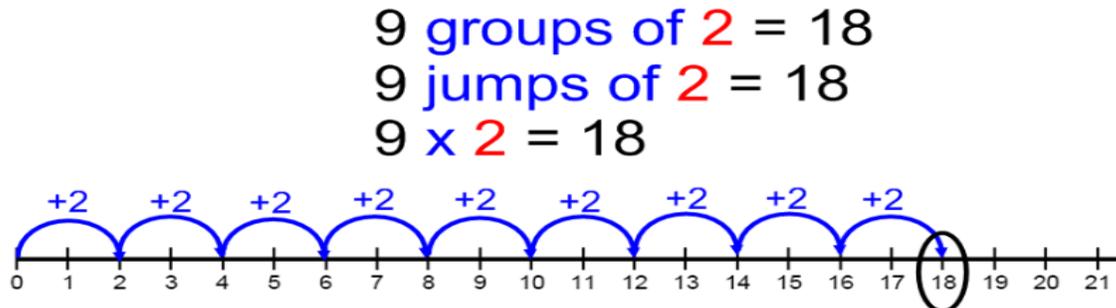


Use arrays



Recall multiplication and division facts for 2, 5, 10

Model and bridge link from repeated addition to solving multiplication problems using a number line.

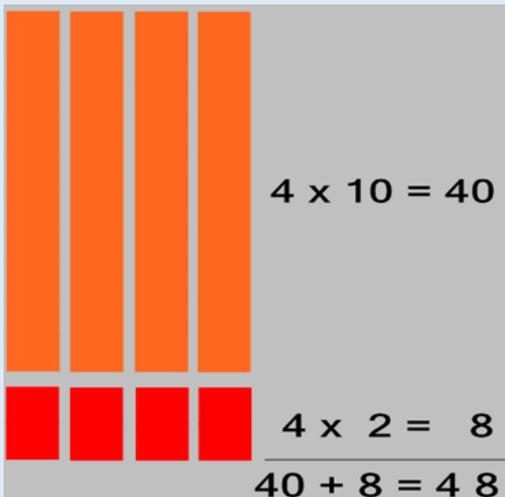




Develop recall of multiplication facts (alongside inverse of the corresponding division facts).

x	3	4	8	x	4	?	?
5				?	8	6	10
6				6	24	18	30
4				?	32	24	40

$12 \times 4 = 48$



Multiplication: Year 3



Year 3 statutory requirements:

- ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- ✓ Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- ✓ Solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Use concrete resources and pictorial representations to develop conceptual understanding of the grid method of multiplication.

x	10	2
4		

OR

x	10	2
4	40	8



The yellow ribbon is 4 times as long as the red ribbon. What is its length?

Multiplication: Year 4

Year 4 statutory requirement:

- ✓ Recall multiplication and division facts for multiplication tables up to 12×12
- ✓ Use place value, known and derived facts to multiply and divide mentally, including: multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- ✓ Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Build on learning from Year 3, continue to use the grid method to multiply increasingly larger numbers.

$$18 \times 3 = 54$$

X	10	8
3	30	24

$$135 \times 5 = 675$$

X	100	30	5
5	500	150	25

$$24 \times 16 = 384$$

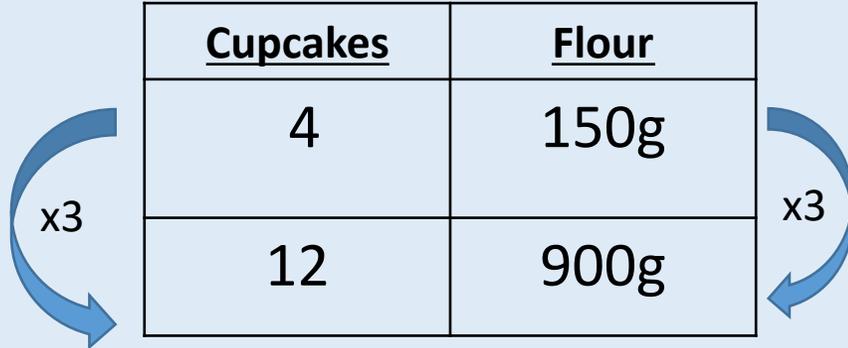
X	20	4
10	200	40
6	120	24

Develop recall of multiplication facts (alongside the inverse of the corresponding division facts).

2 eggs
150g flour
180g sugar

Use knowledge of times tables to solve scaling problems.

Susie wants to bake 12 cupcakes people.
The ingredients given are for four cupcakes.
How much flour will she need?



Multiplication: Year 5

Year 5 statutory requirements:

- ✓ Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- ✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

To multiply by 10, 100, 1000 children should use place value charts to show that the digit moves a column (s) to the left. The value of the digit is increasing by 10, 100 or 1000 times.

Progressing from grid method to short multiplication The short multiplication method is introduced alongside the grid method and the expanded form to aid understanding.

24 x 6 =

20	4
6	24

→

24	
x 6	
24	(4x6)
120	(20x6)
144	

→

24	x 6	=
144		

Place value chart showing 36.42. Columns: Th, H, T, U, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$.

X 10

Place value chart showing 364.2. Columns: Th, H, T, U, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$.

124 x 5 =

100	24
x 5	
500	
100	120
620	

→

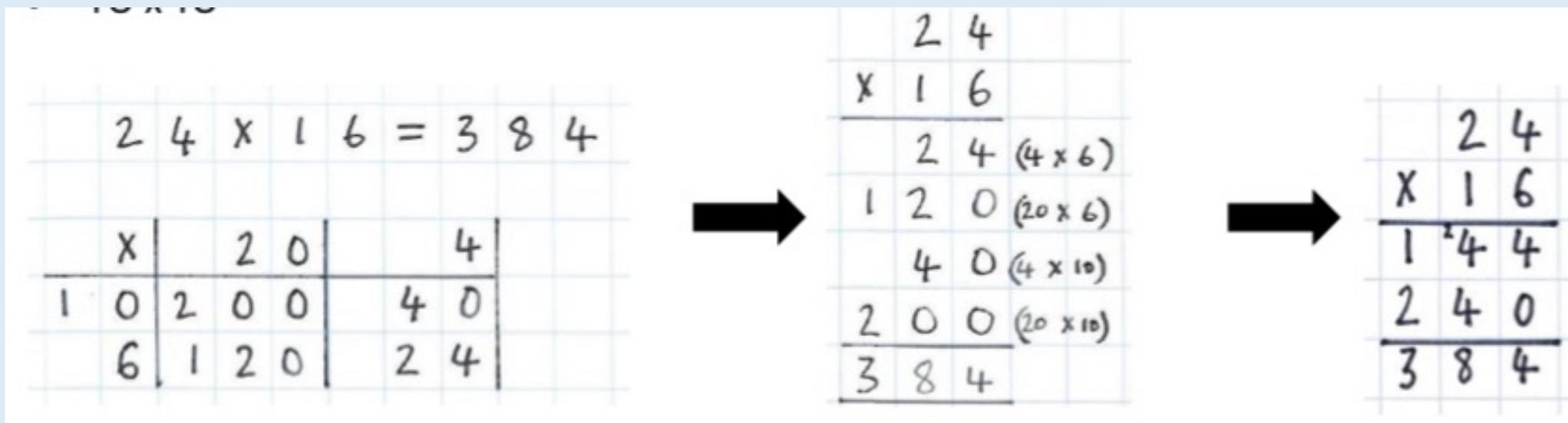
124	x 5	=
620		

Multiplication: Year 5

Year 5 statutory requirements:

- ✓ Multiply numbers up to 4 digits by a 1- or 2-digit number using a formal written method, including long multiplication for two-digit numbers.
- ✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Progressing from grid method to long multiplication. When long multiplication is first introduced, both equations should be presented so that the answers to the individual multiplication steps are on the same line. Children should be encouraged to discuss what is similar and what is different.



The diagram illustrates the transition from a grid method to a formal long multiplication method for the equation $24 \times 16 = 384$.

Grid Method (Left): The equation is written as $24 \times 16 = 384$. Below it, a grid is used to calculate the product. The grid is divided into three columns by a vertical line and two horizontal lines. The top row contains the numbers 2, 0, and 4. The left column contains the numbers 1, 0, and 6. The bottom row contains the numbers 2, 0, and 0. The right column contains the numbers 4, 0, and 4. The final result, 384, is written in the bottom-right cell.

Long Multiplication (Middle): The equation is written as $24 \times 16 = 384$. Below it, the long multiplication method is shown. The numbers 24 and 16 are written vertically. The product is calculated in three steps: $4 \times 6 = 24$, $20 \times 6 = 120$, and $4 \times 10 = 40$. The final result, 384, is written at the bottom.

Formal Long Multiplication (Right): The equation is written as $24 \times 16 = 384$. Below it, the formal long multiplication method is shown. The numbers 24 and 16 are written vertically. The product is calculated in three steps: $4 \times 6 = 24$, $20 \times 6 = 120$, and $4 \times 10 = 40$. The final result, 384, is written at the bottom.

Multiplication: Year 6

Year 6 statutory requirements:

- ✓ Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- ✓ Multiply one-digit numbers with up to two decimal places by whole numbers.

Continue to use compact short multiplication to multiply by single digits

$$\begin{array}{r}
 342 \\
 \times 7 \\
 \hline
 2394 \\
 \hline
 21
 \end{array}$$

$$\begin{array}{r}
 2741 \\
 \times 6 \\
 \hline
 16446 \\
 \hline
 42
 \end{array}$$

Multiplying a two digit number by a three or four digit number should be introduced through the grid method before moving to long multiplication to aid understanding. When long multiplication is first introduced, both equations should be presented so that the answers to the individual multiplication steps are on the same line. Children should be encouraged to discuss what is similar and what is different.

262 x 19 = 4,978

x	200	60	2	
10	2,000	600	20	2620
9	1,800	540	18	2358
				<u>4,978</u>

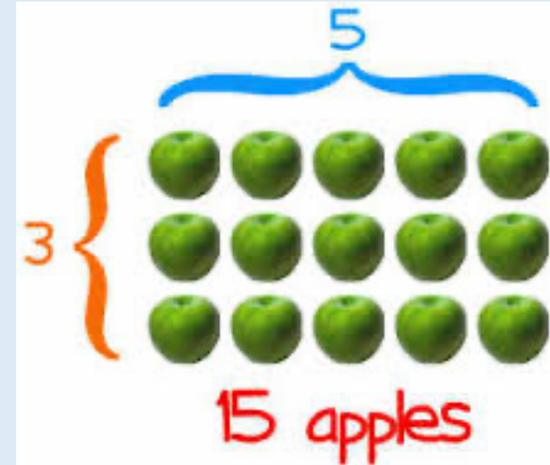
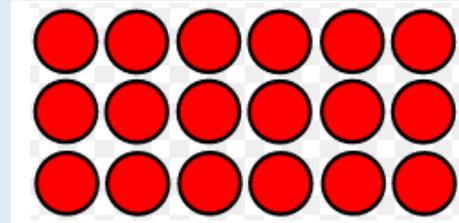
→

	262	
x	19	
	<u>2358</u>	
	2620	
	<u>4,978</u>	

Division

Concrete resources:

- Arrays
- Multiplication squares
- 100 square
- Number lines
- Blank number lines
- Counting stick
- Place value apparatus

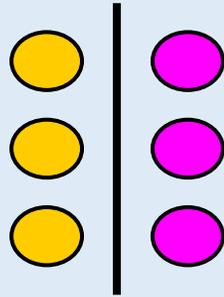
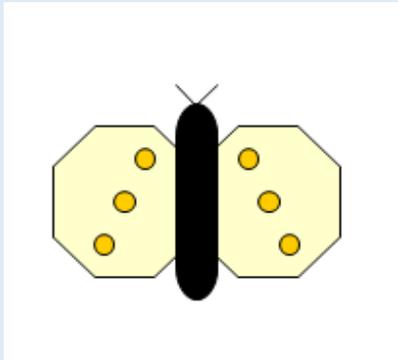


divided by group
 into lots of $\frac{\bullet}{-}{\bullet}$ into groups of
 divisible remainder halve
 half factor

Division: Reception

Early learning goal statutory requirement:
✓ They solve problems, including halving and sharing.

Use pictorial representations and concrete resources to halve numbers to 10.



Begin to share quantities using practical resources, role play, stories and songs.



Role play example:
It is the end of the party and the final two teddies are waiting for their party bags. Provide empty party bags and a small collection of items such as gifts, balloons and slices of cake. Ask the children to share the objects between the two bags.

Division: Year 1

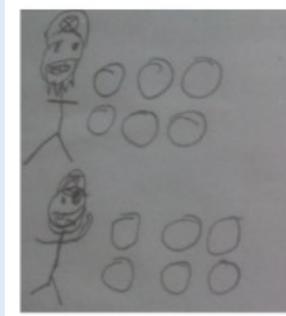
Year 1 statutory requirement:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

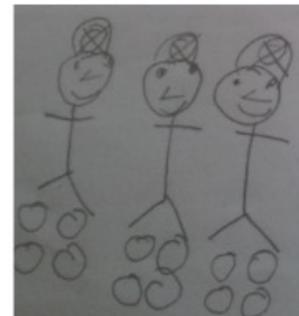
Understand division as **sharing** using concrete resources.



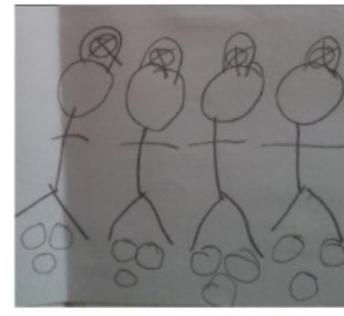
Pictorial representation of sharing **12 gold coins** between 2, 3 and 4 pirates!



$$12 \div 2$$



$$12 \div 3$$



$$12 \div 4$$

Begin to understand division as **grouping** using concrete resources.

12 into groups of 2

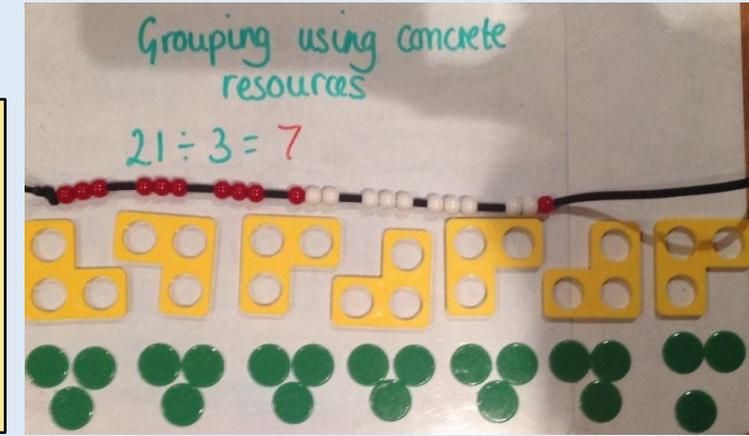
$$12 \div 2 = 6$$



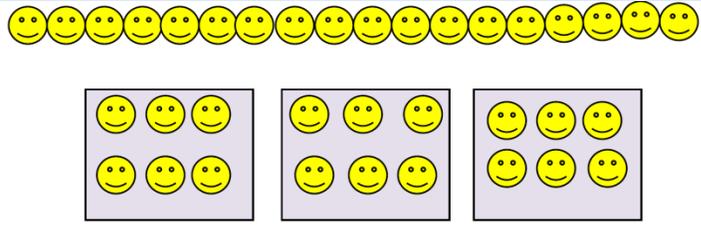
Division: Year 2

Year 2 statutory requirement:

- ✓ Recall and use division facts for 2, 5 and 10 multiplication tables.
- ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write then using the multiplication (x), division (÷) and equals (=) signs.
- ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
- ✓ Find $\frac{1}{3}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{3}{4}$ of a length, shape, set of objects or quantity

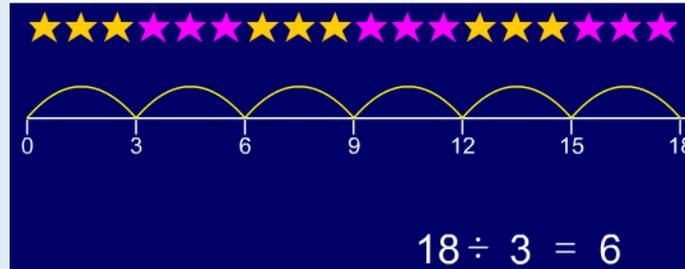


Further develop understanding of difference between **sharing** and **grouping** using concrete resources.



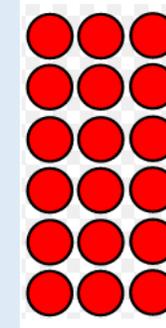
18 smiley faces shared between 3 classes.

Model division as grouping on a number line (ITP 'Grouping')



Children use numbered number lines to divide using grouping.

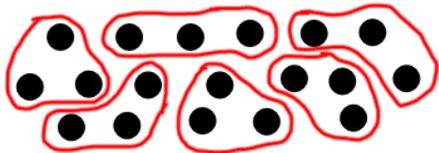
Reinforce division through the use of arrays.



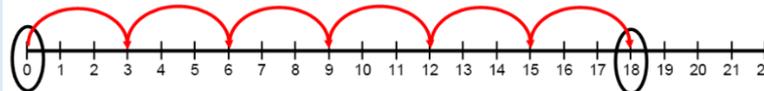
$$18 \div 3 = 6$$

$$18 \div 6 = 3$$

18 into groups of 3
 $18 \div 3 = 6$



18 into groups of 3 = 6 groups
18 into jumps of 3 = 6 jumps
 $18 \div 3 = 6$



Remember to develop connections between fractions and division and rephrase this calculation as $\frac{1}{3}$ of 18 is the same as $18 \div 3 = 6$.



Division: Year 3 & 4



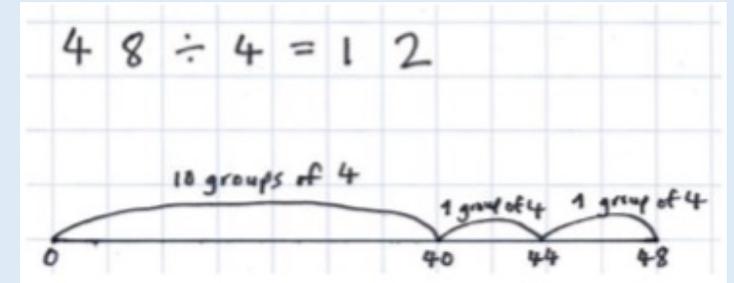
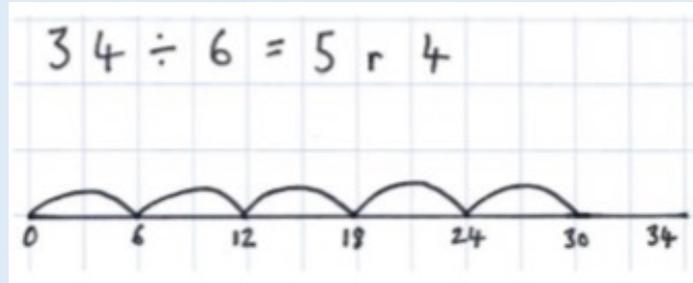
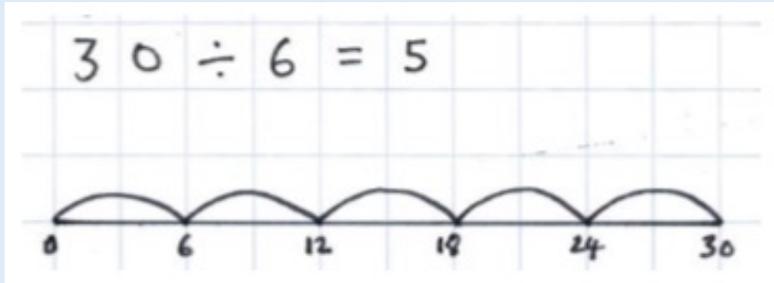
Year 3 statutory requirement:

- ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- ✓ Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- ✓ Solve problems, including missing number problems, involving division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

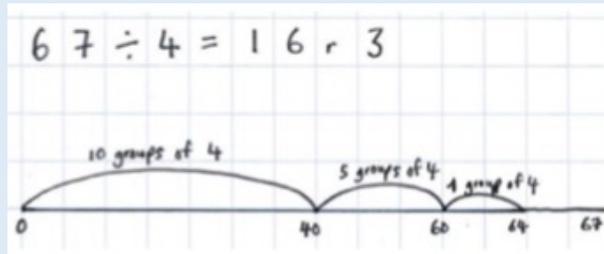
1. Grouping: A number line counting up from zero. This representation should be supported by grouping of concrete materials and other pictorial representations.

2. Grouping with remainders: Again using a number line counting up from zero. This should also be supported by grouping of concrete materials and other pictorial representations.

3. Efficient Grouping: Rather than counting individually, children now use groups for efficiency. The number of groups should be recorded above the jump.



4. Efficient Grouping with remainders: The efficient grouping method now incorporates remainders



Division: Year 3 & 4*



Year 4 statutory requirement: **Note** - there isn't a statutory objective for division. However, Y4 statutory multiplication objectives are to (1) recall multiplication and division facts for multiplication tables up to 12×12 and (2) multiply two-digit and three-digit numbers by a one-digit number using formal written layout so we will build on the connections between multiplication and division.

In Year 4, continue to develop the use of informal number line methods taught in Year 3. Start to teach compact short division when children are secure with dividing on a number line. Teach compact short division using the following progression.

See video link in 'notes' to consider how to develop conceptual understanding of division using dienes.

$$\begin{array}{r} 32 \\ 3 \overline{) 96} \end{array}$$

Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor).

$$\begin{array}{r} 18 \\ 4 \overline{) 732} \end{array}$$

Limit numbers to NO remainders in the final answer, but with remainders occurring within the calculation.

$$\begin{array}{r} 218 \\ 4 \overline{) 8732} \end{array}$$

Extend to 3-digit number first where the divisor can go into the first number and then progress to when the divisor cannot go into the first number.

$$\begin{array}{r} 037 \\ 5 \overline{) 1835} \end{array}$$

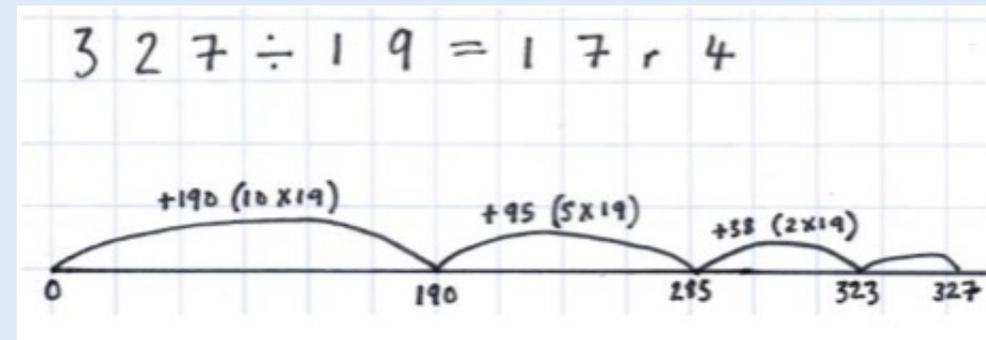
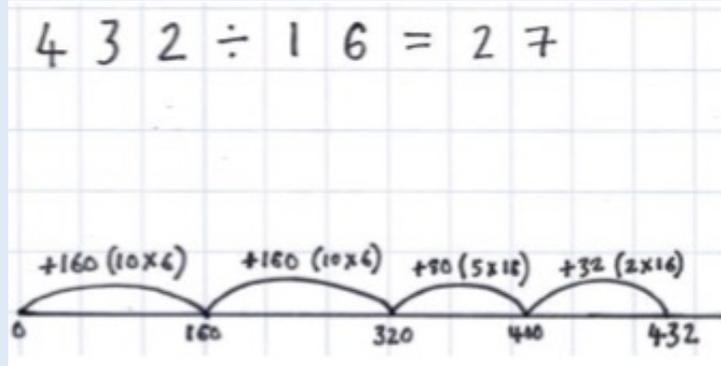
Remember to develop connections between fractions and division and rephrase these calculations as $\frac{1}{3}$ of 96; $\frac{1}{4}$ of 72, $\frac{1}{4}$ of 872 and $\frac{1}{5}$ of 185. Note: Year 3 fraction objective - *Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators*; Year 4 fraction objective: *solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.*



Division: Year 5

Continue to use number lines to support the children's understanding of efficient grouping or the 'chunking' of multiples of the divisor.

Year 5 statutory requirement:
 ✓ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.



Further secure pupils' understanding of compact short division.

$$218 \div 8 = 27 \text{ r } 2$$

$$8 \overline{) 2158}$$

$96 \div 4 = 24$

$$\begin{array}{r} 24 \\ 4 \overline{) 96} \\ \underline{-40} (10 \times 4) \\ 56 \\ \underline{-40} (10 \times 4) \\ 16 \\ \underline{-16} (4 \times 4) \\ 00 \end{array}$$

The vertical chunking method is introduced but only with a single digit divisor. The number of groups should be recorded alongside on the right with the answer written on top of the bus stop. When this is secure, the same layout is used with remainders.

$43 \div 3 = 14 \text{ r } 1$

$$\begin{array}{r} 14 \text{ r } 1 \\ 3 \overline{) 43} \\ \underline{-30} (10 \times 3) \\ 13 \\ \underline{-12} (4 \times 3) \\ 01 \end{array}$$

Extend to expressing results in different ways according to the context, including with remainders as fractions, as decimals or by rounding. For example:

- Whole number remainder = $27 \text{ r } 2$
- Fraction remainder = $27 \frac{2}{8} = 27 \frac{1}{4}$
- Decimal remainder = $27 \frac{1}{4} = 27 \frac{25}{100} = 27.25$



Division: Year 6

Year 6 statutory requirement:

✓ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Continue to use compact short division to divide numbers up to 4 digits by a 1-digit whole number.

Continue to develop the formal 'chunking' method to divide 3- or 4-digit numbers by a 2-digit whole number.

$$218 \div 8 =$$

$$27 \text{ r } 2$$

$$8 \overline{) 2158}$$

- Whole number remainder = **27 r 2**
- Fraction remainder = $27 \frac{2}{8} = 27 \frac{1}{4}$
- Decimal remainder = $27 \frac{1}{4} = 27 \frac{25}{100} = 27.25$

$$432 \div 16 = 27$$

$$\begin{array}{r} 27 \\ 16 \overline{) 432} \\ \underline{-160} \quad (10 \times 16) \\ 272 \\ \underline{-160} \quad (10 \times 16) \\ 112 \\ \underline{-80} \quad (5 \times 16) \\ 32 \\ \underline{-32} \quad (2 \times 16) \\ 00 \end{array}$$

$$327 \div 19$$

$$\begin{array}{r} 17 \text{ r } 4 \\ 19 \overline{) 327} \\ \underline{-190} \quad (10 \times 19) \\ 137 \\ \underline{-95} \quad (5 \times 19) \\ 42 \\ \underline{-38} \quad (2 \times 19) \\ 04 \end{array}$$

$$\begin{array}{r} 23 \\ 19 \overline{) 437} \\ \underline{-38} \quad \downarrow \\ 057 \\ \underline{-57} \\ 00 \end{array}$$

$$\begin{array}{r} 024 \text{ r } 12 \\ 24 \overline{) 588} \\ \underline{-48} \\ 108 \\ \underline{-96} \\ 12 \end{array}$$

If appropriate, the formal long division method can be introduced to divide numbers up to 4 digits by a 2-digit whole number.